Synthesis and Realization of Narrow-Band Canonical Microwave Bandpass Filters Exhibiting Linear Phase and Transmission Zeros

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Invited Paper

Abstract —The present study is concerned with the synthesis and realization of narrow-band canonical microwave bandpass filters exhibiting additional couplings between nonsuccessive resonant circuits. These bypass couplings effect, on the one hand, attenuation poles at finite frequencies and, on the other hand, make the bandpass filter nonminimum-phase, and therefore offer degrees of freedom for group delay equalization in the passband. The realization of canonical network structures with bypass couplings is possible by extracting from the chain matrix an ideal transformer, the input of which is connected, for example, in shunt with and its output in series with the remainder two-port. The extraction is associated with the reduction by 2 of the degree of the denominator polynomial of the chain matrix elements. To realize a clear correlation between the bypass elements and the electrical performance, canonical structures with sections that are bypassed only once or twice are proposed. Theoretical and measured curves of the examples of realization for 1235 MHz, consisting of coaxial cavities, and for 4000 MHz, consisting of dual-mode cavities, are seen to be in close agreement.

I. Introduction

THE VIGOROUS development of communication satellites prompted in the early 1970's increased worldwide efforts to realize satellite subsystems in which compact design, low weight, insensitivity to temperature, and functional reliability are accompanied by outstanding electrical performance. Special attention was also devoted to the creation of electrical microwave filters as integral elements of transponders, with the aid of new mathematical approaches, synthesis techniques, and methods of realization leading to high return loss, minimum attenuation distortion, and, in some cases, a flat group delay (linear phase) in the passband and high attenuation in the stopbands. The demand for compact and functionally reliable design was, however, incompatible in those days with the common practice of realizing delay equalization by inserting a delay equalizer, consisting of a circulator or a 3-dB directional coupler and reactance networks, behind the minimum-phase filter.

The bulky and inhomogeneous design of such configurations, their large number of connecting points, and high production cost are only some of the arguments against trying to satisfy the foregoing demands by such means. Thus, an alternative solution was sought in which delay equalization is realized within the filter itself.

Circuit configurations containing all-pass networks composed of lumped elements as reported in those days, for example, by Hibino *et al.* [1] and Stagg [2] are little amenable to conversion into suitably realizable microwave structures. (A method of synthesizing microwave bandpass filters with all-pass networks proposed by Rhodes and Ismail [3] in 1972 leads to mechanically elaborate and bulky waveguide structures [4].)

It could be said in retrospect that the key to the solution of the described filter problem lay, in a general sense, in the controlled coupling of nonsuccessive filter elements. The additional degrees of freedom thereby realized allow, depending on the sign of the bypass elements, either a linearization of phase (group delay equalization) or the realization of attenuation poles at finite frequencies. Both effects are also simultaneously realizable in a filter network [5].

The first reference to the possibility of coupling nonsuccessive filter resonant circuits by way of transmission lines was, in our recollection, already published by Dishal [6] in 1956, but attracted little interest on account, presumably, of the bulky layout resulting from the use of external coupling transmission lines. Compact filter designs can be realized if the cavities are arranged to allow the direct coupling of those that are to be additionally coupled. Almost all the realizations of bandpass filters with bypass elements that have been reported since are based on this principle. Kurzrok, as the first user of this procedure, introduced in 1966 a four-cavity inductively coupled microwave bandpass filter [7] in which cavities 1 and 4 were additionally capacitively coupled in order to realize a sharper cutoff for each of the two stopbands by a respective attenuation pole. A detailed historical survey of the development of linear phase filters up to 1976 was presented by Levy [8]. Numerous detailed reports on linear phase filters have been presented in particular by Rhodes and subsequently also by Atia and Williams. The pioneer reports by Rhodes [9]–[11] appeared in June 1970.

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Similar investigations [12] conducted around the same time at the one-time Central Communications Laboratory of the Siemens AG in Munich, ultimately led to an extremely flexible method of synthesis for bypassed filter networks that was reported in October 1970 [5]. The synthesis described below starts, as is customary in insertion-loss theory, with the characteristic function $K(s)$, which can be selectively constructed in correspondence with the requirements (Chebyshev return loss response and delay equalization within the passband, use of attenuation poles at finite frequencies) to allow the synthesis of canonical and symmetrical or unsymmetrical networks. The exigencies imposed by the limited possibilities of realization of microwave bandpass filters can already be taken into account when designing the reference low-pass filter and coordinated with the electrical specifications. Various realizations of narrow-band microwave bandpass filters with bypass elements demonstrate the universality of the method.

II. Characteristic Function and Chain Matrix

The reference low-pass network is characterized by its chain matrix $[A(s)]$. The output of the network is terminated into the ohmic load resistance R_2, while its input is driven by a voltage source V_0 with the internal ohmic resistance R_1. Using normalized impedances, we assume without restriction of generality $R_1 = R_2 = 1$. The passband of all the reference low-pass filters used in the present paper shows a Chebyshev return-loss response with a specifiable ripple level; the low-pass frequency variable $s = \sigma + j\Omega$ is normalized to the edge of the passband (Fig. 3). The relationship with the design data of the microwave bandpass filter—the center frequency f_0 and relative passband width B— is given by the conventional frequency transformation

$$\Omega = \frac{1}{B}\left(\frac{f}{f_0} - \frac{f_0}{f}\right). \tag{1}$$

The group delay response is then computed with

$$\tau(f) = \frac{T(\Omega)}{Bf_0}\left[1 + \left(\frac{f_0}{f}\right)^2\right] \tag{2}$$

the normalized group delay of the reference low-pass filter being defined by

$$T(\Omega) = \frac{1}{2\pi}\frac{db(\Omega)}{d\Omega} \tag{3}$$

where b is the phase of the reference low-pass filter. The transfer function is generally known to be defined by

$$F(s) = 2\frac{V_2}{V_0}\sqrt{\frac{R_1}{R_2}} = c_2\frac{f(s)}{g(s)} \tag{4}$$

where $g(s)$ represents a strict Hurwitz polynomial, whereas the numerator polynomial $f(s)$ for the case of a low-pass filter is an even polynomial in s.

As is customary in insertion-loss theory [13], the reference low-pass filter is designed by way of the characteristic function

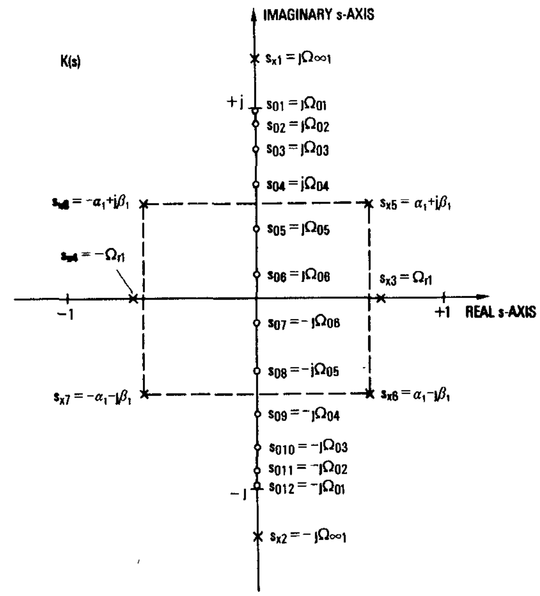

Fig. 1. Position of zeros and poles of $K(s)$ indicated for the example of a prototype low-pass filter the responses of which are shown in Fig. 3. $s_{01} \cdots s_{012}$: zeros on the imaginary s-axis $\rightarrow \prod_{\nu=1}^{6}(s^2 + \Omega_{0\nu}^2)$; s_{x1}, s_{x2}: poles on the imaginary s-axis $\rightarrow (s^2 + \Omega_{\infty 1}^2)$; s_{x3}, s_{x4}: poles on the real s-axis $\rightarrow (s^2 - \Omega_{r1}^2)$; $s_{x5} \cdots s_{x8}$: pole quadruplet $\rightarrow (s^4 + M_1 s^2 + N_1)$.

$$K(s) = \frac{c_1 h(s)}{|c_2| f(s)} = \frac{c_1 \prod_{\nu=1}^{n}(s - s_{0\nu})}{|c_2| \prod_{\nu=1}^{2m}(s - s_{x\nu})}$$

$$= \frac{c_1}{|c_2|}\frac{s^\gamma \prod_{\nu=1}^{\nu_1}\left(s^2 + \Omega_{0\nu}^2\right)}{\prod_{\nu=1}^{\nu_2}\left(s^2 + \Omega_{\infty\nu}^2\right)\prod_{\nu=1}^{\nu_3}\left(s^2 - \Omega_{r\nu}^2\right)\prod_{\nu=1}^{\nu_4}\left(s^4 + M_\nu s^2 + N_\nu\right)},$$

$$c_1 = \pm 1 \quad M_\nu = 2\left(\beta_\nu^2 - \alpha_\nu^2\right)$$

$$N_\nu c = \left(\alpha_\nu^2 + \beta_\nu^2\right)^2 \tag{5}$$

from which the attenuation can be calculated as follows:

$$a_B = 10\log\left(1 + |K(j\Omega)|^2\right) \text{ in dB}. \tag{6}$$

Fig. 1 shows the position of the zeros and poles of $K(s)$ for the example of the reference low-pass filter described by (13). It is practical to place zeros of $h(s)$ in pairs on the imaginary s-axis within the interval $-j < s_{0\nu} < +j$ so that the zeros of the attenuation will appear at the frequencies $\Omega_{0\nu}$ in the passband $0 \leqslant \Omega < 1$. Then, $h(s)$ is either an even or an odd polynomial in s; γ should assume (preferably) only the value 0 or 1. For the appearance of poles at finite frequencies $\Omega_{\infty\nu} > 1$, the zeros of $f(s)$ appear in pairs on the imaginary s-axis. Zeros of $f(s)$ that do not appear on the imaginary s-axis are referred to below as complex poles. The inclusion of complex poles in the characteristic function results in additional parameters, given by the position of these poles in the complex s-plane, that can be used for delay equalization with a chosen passband and stopband attenuation response. The complex poles lie, as shown in Fig. 1, either in quadruplets in the s-plane, characterized by the real part $\pm\alpha_\nu$ and the imaginary part $\pm\beta_\nu$, or at $s=\pm\Omega_{r\nu}$ on the real s-axis.

The number q of attenuation poles at the frequency $\Omega=\infty$ results as the difference of the polynomial degrees of $h(s)$ and $f(s)$

$$q=2\nu_1+\gamma-2m\geq 1, \qquad \text{with } m=\nu_2+\nu_3+2\nu_4. \quad (7)$$

Due to the presence of the poles of $K(s)$ in the right s-half plane, a nonminimum phase low-pass filter results. The group delay is calculated from the Hurwitz polynomial $g(s)$ derived from the characteristic function $K(s)$ by way of the principal equation of insertion-loss theory

$$g(s)g(-s)=h(s)h(-s)+c_2^2f(s)f(-s). \quad (8)$$

The characteristic function $K(s)$ can be optimized with respect to return loss, group delay (via $g(s)$), and attenuation by means of known procedures, e.g., [14]–[16], after having estimated the required filter order n with the aid of reference tables [17], [18]. An optimization example is presented in Fig. 3.

The chain matrix $[A(s)]$ of the reference low-pass filter is obtained in the conventional manner [13] from the polynomials $g(s)$, $c_1h(s)$, and, $|c_2|f(s)$. The abridgments $EV\{\ \}$, $OD\{\ \}$ denote the respective even and odd parts of the polynomial in curly brackets. The polynomial $f(s)$ is assumed to be an even polynomial in s

$$[A(s)]=\begin{bmatrix}A_{11}(s) & A_{12}(s)\\ A_{21}(s) & A_{22}(s)\end{bmatrix}=\frac{1}{G(s)}\begin{bmatrix}G_1(s) & U_1(s)\\ U_2(s) & G_2(s)\end{bmatrix} \quad (9)$$

$$\begin{aligned}G(s)&=|c_2|f(s)\\ G_1(s)&=\mathrm{EV}\{g(s)\}+c_1\mathrm{EV}\{h(s)\}\\ U_1(s)&=\mathrm{OD}\{g(s)\}+c_1\mathrm{OD}\{h(s)\}\\ U_2(s)&=\mathrm{OD}\{g(s)\}-c_1\mathrm{OD}\{h(s)\}\\ G_2(s)&=\mathrm{EV}\{g(s)\}-c_1\mathrm{EV}\{h(s)\}.\end{aligned} \quad (10)$$

$G(s)$, $G_1(s)$, $G_2(s)$ are even, while $U_1(s)$ and $U_2(s)$ are odd polynomials in s, $c_1=\pm 1$.

III. Low-Pass Filter Synthesis

The desired extraction of bypass elements is possible with the aid of an equivalence given by Cauer [19] based on the consideration of positive functions and depicted in Fig. 2. Accordingly, from the circuit design aspect, the transition from the two-port TP to the two-port TP^* is effected by connecting an ideal transformer with the turns ratio $1:k$, such that it lies in shunt with the primary port and in series with the second port of the two-port TP^*. The relation between the chain matrices of the two-ports TP and TP^* are given by

$$\begin{aligned}G^*(s)&=G(s)-kG_1(s)\\ G_1^*(s)&=G_1(s)\\ U_1^*(s)&=U_1(s)\\ U_2^*(s)&=U_2(s)\\ G_2^*(s)&=G_2(s)-2kG(s)+k^2G_1(s).\end{aligned} \quad (11)$$

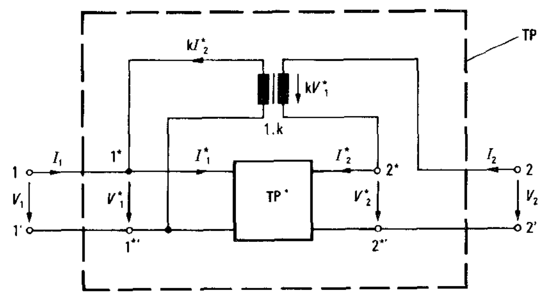

Fig. 2. Decomposition of an ideal transformer from the two-port TP.

If the polynomials $G(s)$ and $G_1(s)$ are of the same degree, it will be possible to choose the constant k such that the coefficients of the respectively highest power p of the two polynomials will have the same value. This leads to the condition

$$k=\frac{G_p}{G_{1,p}} \quad (12)$$

for a reduction of the degree of the polynomial $G(s)$ by 2. The extraction of a bypassing ideal transformer with the turns ratio $1:k$ from the two-port TP (according to Fig. 2) then corresponds, for example, to the removal of an attenuation pole at the finite frequency Ω_∞. The chain matrix $[A^*(s)]$ of the remaining two-port TP^* is defined by the relations (11) and serves for calculating the remaining circuit elements of the low-pass filter.

The reference low-pass network is synthesized according to the well-known methods of insertion-loss theory by extracting circuit elements from the chain matrix. Besides the usual elements, such as inductors, capacitors, and ideal transformers, the "bypassing ideal transformer" described in the preceding section will here be specially used for synthesis.

The synthesis procedure generally leads to a ladder network consisting of shunt capacitances and series inductances. The number of these reactances corresponds to the order n of the chain matrix. The reduction of the degree of the polynomial $G(s)$ in steps of 2 causes an inductance at the one actual end and a capacitance at the other actual end of the structure to be coupled per step by means of a bypassing ideal transformer until ultimately a network with an m-times bypassed section results. Since extraction is effected from the two actual ends of the network alternately, the extraction process ends with an ideal transformer in the interior of the network. Since there can only be as many inductors, capacitors, and ideal transformers as the characteristic function has coefficients or free parameters, the networks are canonical.

The synthesis of low-pass filters with bypass elements according to the proposed method will be described below for a reference low-pass network of the order $n=12$ that will also be used as prototype for one of the examples of realization to be described later. As shown in Fig. 3 the attenuation response has been steepened by a pole at $\Omega=1.26$. In the passband $0\leqslant\Omega\leqslant 1$, the return loss is $a_r\geqslant 28$ dB, while a pole pair on the real s-axis as well as a

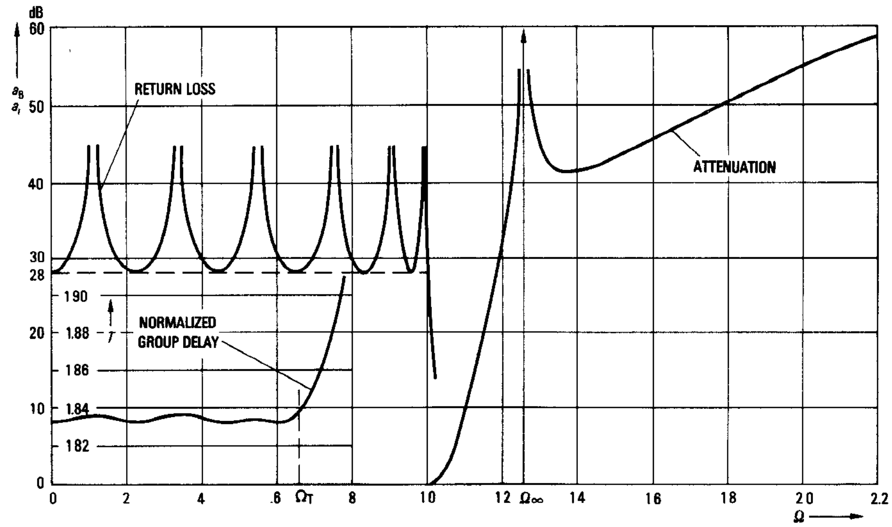

Fig. 3. Theoretical responses of the return loss, attenuation, and group delay of the nonminimum-phase low-pass filter (14) of the order $n=12$ exhibiting one attenuation pole at $\Omega=1.26$.

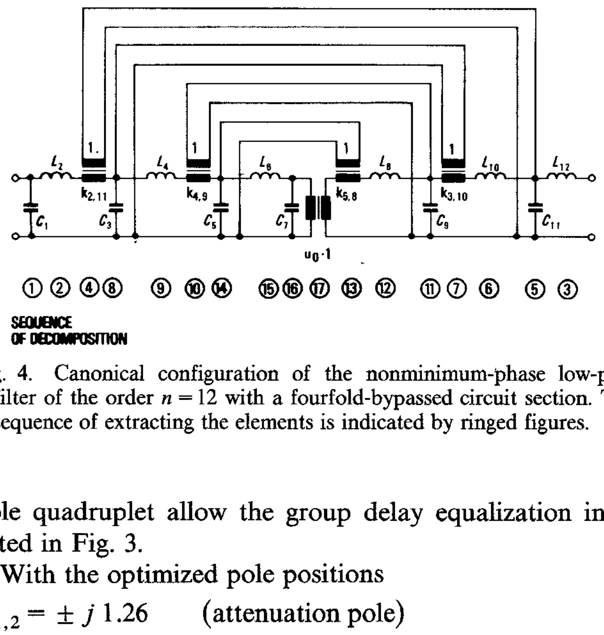

Fig. 4. Canonical configuration of the nonminimum-phase low-pass filter of the order $n=12$ with a fourfold-bypassed circuit section. The sequence of extracting the elements is indicated by ringed figures.

pole quadruplet allow the group delay equalization indicated in Fig. 3.

With the optimized pole positions

$$
\begin{aligned}
s_{x1,2} &= \pm j\,1.26 \quad \text{(attenuation pole)}\\
s_{x3,4} &= \pm 0.66036456 \quad \text{(pole pair on real } s\text{-axis)}\\
s_{x5,6,7,8} &= \pm 0.59843434 \pm j\,0.51255268 \quad \text{(pole quadruplet)}
\end{aligned}
\tag{13}
$$

the following expressions are obtained for the chain matrix polynomials ($c_1=-1$; $c_2=0.018427665$):

$$
\begin{aligned}
G(s) &= 0.018427665\,s^8 + 0.017703307\,s^6 - 0.0097045362\,s^4 + 0.010613396\,s^2 - 0.0049173349\\
G_1(s) &= 3.2622780\,s^{10} + 8.6008486\,s^8 + 7.6877506\,s^6 + 2.6633024\,s^4 + 0.29770910\,s^2 + 0.0047253184\\
U_1(s) = U_2(s) &= 2.5543210\,s^{11} + 9.0185411\,s^9 + 11.151100\,s^7 + 5.7514892\,s^5 + 1.1318889\,s^3 + 0.056665784\,s\\
G_2(s) &= 2\,s^{12} + 8.8498899\,s^{10} + 14.348562\,s^8 + 10.343627\,s^6 + 3.1979783\,s^4 + 0.33504798\,s^2 + 0.0051171542.
\end{aligned}
\tag{14}
$$

The synthesis steps for the example are described below in detail and listed in Table I; the resulting low-pass network is shown in Fig. 4. Assuming the characteristic function $K(s)$, and, consequently, the chain matrix $[A(s)]$, to have $q \geqslant 1$ poles at $s=\infty$, the special example will have $q=4$. From these, $(q-1)=3$ poles will first be removed as C_1, L_2, and L_{12}. To obtain L_{12}, the direction of removal must be changed; in the chain matrix this results solely in the positions of $G_1(s)$ and $G_2(s)$ being interchanged. With polynomials $G(s)$ and $G_1(s)$ of the same degree it is possible, with attention paid to the direction of removal, to take account of the bypassing ideal transformer $k_{2,11}$. The resulting reduction of the degree of the polynomial $G(s)$ by 2 releases two additional poles at $s=\infty$, which can consequently be realized, again paying attention to the direction of removal, as C_{11} and L_{10}. Thus, the extraction of the bypassing ideal transformer can be regarded as a partial removal through which two complex poles shift to $s=\infty$ and may be interpreted as a shunt capacitor and a series inductor. The three extraction steps form a cycle which, as may be seen from Table I, is repeated after each changing of the direction of removal until the degree of the polynomial $G(s)$ has become zero. Altogether, there are m such extraction cycles. As the last reactance it is necessary to take account of an initially reserved pole at $s=\infty$ (as C_7). Thus, all chain matrix polynomials are of zero degree and the extraction process ends at the ideal transformer u_0 in the interior of the circuit.

The low-pass includes a section that is bypassed by four coupling elements. These are collectively responsible for both the group delay response and the attenuation pole at $\Omega=1.26$, which means that it is not possible to establish any clear correlation between a particular bypass element

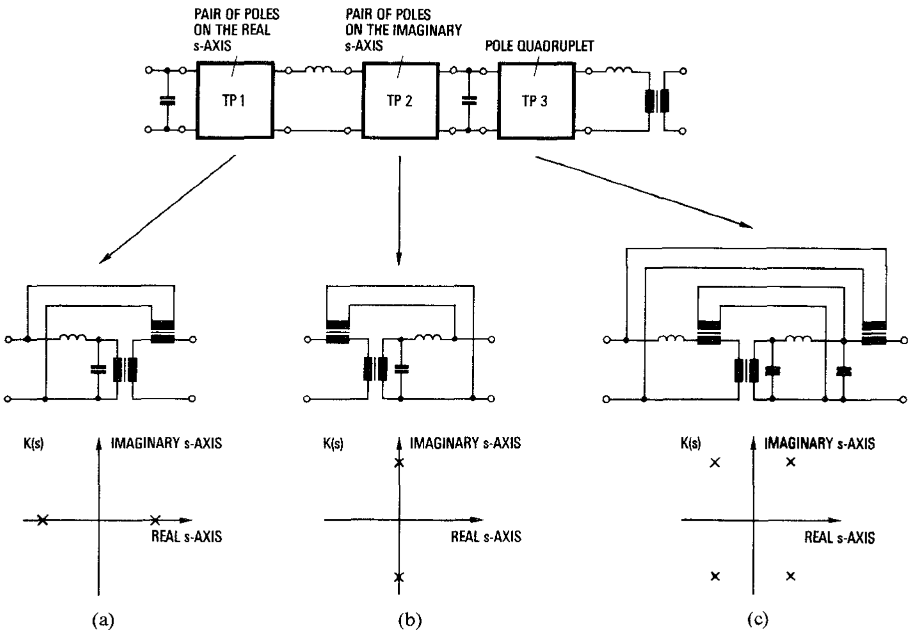

Fig. 5. 12th-order low-pass configuration containing three circuit blocks, each realizing (a) a pole pair on the real s-axis, (b) a pole pair on the imaginary s-axis, and (c) a pole quadruplet.

TABLE I
DECOMPOSITION OF THE CHAIN MATRIX (14) OF A 12TH-ORDER NON-MINIMUM-PHASE LOW-PASS FILTER BY MEANS OF BYPASSING IDEAL TRANSFORMERS (THE REMOVAL OF 4 POLES AT $s = \infty$ IS INDICATED BY*)

No.	Direction of removal	Extracted element		Degree of chain matrix polynomials				
				G(s)	$G_1(s)$	$U_1(s)$	$U_2(s)$	$G_2(s)$
		START		8	10	11	11	12
1	→	C_1	•	8	10	11	9	10
2	→	L_2	•	8	8	9	9	10
	←	./.		8	10	9	9	8
3	←	L_{12}	•	8	8	7	9	8
4	←	$k_{2,11}$	I	6	8	7	9	8
5	←	C_{11}		6	8	7	7	6
6	←	L_{10}		6	6	5	7	6
	→	./.		6	6	5	7	6
7	→	$k_{3,10}$	II	4	6	5	7	6
8	→	C_3		4	6	5	5	4
9	→	L_4		4	4	3	5	4
	←	./.		4	4	3	5	4
10	←	$k_{4,9}$	III	2	4	3	5	4
11	←	C_9		2	4	3	3	2
12	←	L_8		2	2	1	3	2
	→	./.		2	2	1	3	2
13	→	$k_{5,8}$	IV	0	2	1	3	2
14	→	C_5		0	2	1	1	0
15	→	L_6		0	0	0	1	0
16	→	C_7	•	0	0	0	0	0
17	→	u_0		0	0	0	0	0

and, for example, the attenuation pole at $\Omega = 1.26$. Establishing a clear correlation would, however, be extremely desirable in order to make the filter easily tunable. One approach to this problem is the factorization of the chain matrix accompanied by the decomposition of the network into individual blocks.

Factorization is effected such [13] that the matrices of the partial twoports have as denominator polynomials the elementary functions $(s^2 + \Omega_\infty^2)$, $(s^2 - \Omega_r^2)$, and $(s^4 + Ms^2 + N)$ of the denominator polynomial $|c_2| f(s) = G(s)$. Various methods are known for the factorization of the chain matrix, e.g., [20]; in the present case, the method described by Hibino *et al.* [1] was used. In this method, the complex poles of a given configuration of poles are individually removed in the form of bypassed T-sections with complex circuit elements. Aggregation results in chain matrices with real coefficients of the order $n = 2$ for pole pairs on the real or imaginary s-axis and others of the order $n = 4$ for pole quadruplets. Each partial chain matrix is decomposed according to the given rules and leads as indicated in Fig. 5 to a block with only one or two bypass elements that now allows a clear correlation between bypass element and pole configuration. The total decomposition of the low-pass network must be controlled in such a way that, as also shown in Fig. 5, each block will be imbedded between two elements of a ladder network composed of shunt capacitors and series inductances.

In the present example, the factorization of the chain matrix (14) effects a circuit structure with three blocks representing a pair of poles on the real s-axis ($s_{x1,2} = \pm 0.66036456$), a pair of poles on the imaginary s-axis ($s_{x3,4} = \pm j\ 1.26$) and a pole quadruplet ($s_{x5,6,7,8} =$

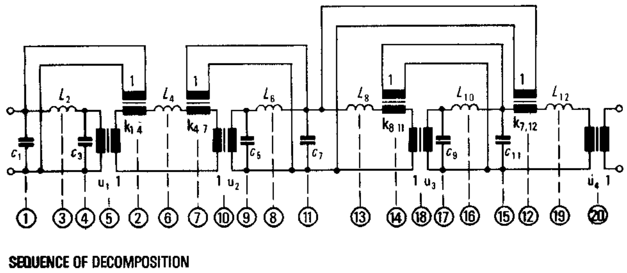

Fig. 6. Canonical low-pass configuration with three circuit blocks. The configuration is equivalent to that depicted in Fig. 4. The sequence of extracting the elements is indicated by ringed figures.

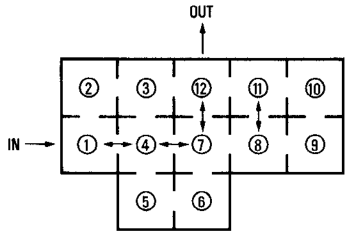

Fig. 7. Schematic arrangement of the 12 resonators of a microwave bandpass filter with three circuit blocks, corresponding to the reference low-pass filter shown in Fig. 6. There are definite correlations between the additional couplings of resonators 1 and 4, 4 and 7, 7 and 12, 8 and 11 and the respective mode of operation.

$\pm 0.59843434 \pm j\, 0.51255268$).

The sequence for the extraction of the circuit elements is listed in Table II; the overall network, which is shown in Fig. 6, is equivalent to the network presented in Fig. 4. Since the four ideal transformers $u_1 \cdots u_4$ can be combined into a single ideal transformer, the network contains 17 elements like the other structure and is therefore canonical. The additional coupling of the elements L_4 and C_4 results in the attenuation pole at $\Omega = 1.26$, whereas the additional couplings of C_1 and L_4, C_7 and L_{12}, L_8 and C_{11} are associated with delay equalization. The element data are also listed in Table II; the negative signs of $k_{1,4}$ and $k_{8,11}$ are explained in the next section.

Fig. 7 shows schematically an appropriate bandpass filter configuration with the additional couplings of cavities 1 and 4, 4 and 7, 7 and 12, 8 and 11.

IV. Conversion of Low-Pass Network and Generation of a Bandpass Equivalent Circuit Configuration

It is practical to convert the synthesized low-pass network by means of useful transformations into an equivalent network that can then be converted by reactance transformation into a corresponding bandpass filter network with bypass elements. The impedance inverters generally used for this purpose consist of an idealized transmission line with the characteristic impedance Z and the frequency-independent phase $b = \pm 90°$. Z can assume the normalized value of unity without loss of validity. First, the series inductances are converted with the aid of two impedance inverters and an ideal transformer with the turns ratio

TABLE II
Sequence of Extracting Elements Demonstrated by the Example of a 12th-Order Low-Pass Filter with Circuit Blocks, the Configuration of which is Shown in Fig. 6

No.	Direction of removal	Extracted element	
1	→	C_1 = 0.78298693	
2	→	$k_{1,4}$ = − .38801225	TP 1
3	→	L_2 = 1.6428798	
4	→	C_3 = 3.5973379	
5	→	u_1 = .72045473	
6	→	L_4 = 1.6454841	
7	←	$k_{4,7}$ = .32431270	TP 2
8	←	L_6 = 2.0158573	
9	←	C_5 = .96346330	
10	←	u_2 = 1.4799745	
11	→	C_7 = 1.9003083	
12	→	$k_{7,12}$ = .13488355	TP 3
13	→	L_8 = 1.7062271	
14	←	$k_{8,11}$ = − .33479248	
15	←	C_{11} = 1.5698756	
16	←	L_{10} = 2.0098566	
17	←	C_9 = 4.7739058	
18	←	u_3 = .67082298	
19	→	L_{12} = .72303126	
20	→	u_4 = − .96095109	

$1:-1$ into shunt capacitors. For the sake of uniformity, all impedance inverters in the basic chain should be assigned respective phases with the same sign, e.g., positive ($b = +90°$). The equivalence indicated in Fig. 8 establishes a correlation between series and shunt ideal bypass transformers; the validity of the equivalent conversion is easily verifiable by shorting or open-circuiting appropriate ports and keeping in mind that the impedance inverter transforms a short into an open circuit and vice versa. If the ideal transformer is interpreted as a change of the characteristic impedance of the adjacent impedance inverter, the result will be a network consisting of n shunt capacitors and of impedance inverters. The latter effect the direct as well as the bypass couplings between the capacitors. Fig. 9 shows the low-pass network in Fig. 6 after equivalent conversion. Formally negative values result for the characteristic impedances $Z_{1,4} = -1/k_{1,4}$ and $Z_{8,11} = -1/k_{8,11}$; since, however, both $k_{1,4}$ and $k_{8,11}$ are themselves negative according to Table II, positive values are also obtained for $Z_{1,4}$ and $Z_{8,11}$. Moreover, due to the interchangeability of the signs for the characteristic impedance and the phase of an impedance inverter, the network can always be controlled such that the characteristic impedances $Z_{\mu,\nu}$ of all impedance inverters $I_{\mu,\nu}$ are positive. If, at the same time, the inverter phases of the basic chain are assumed to be positive, the signs of the phases of the bypassing impedance inverters will disclose directly whether the bypasses are positive or negative. Instead of the terms

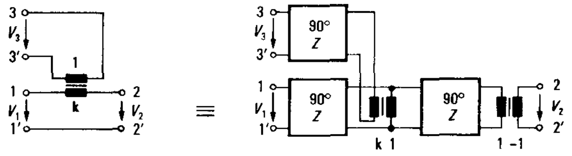

Fig. 8. Equivalent conversion of an ideal series transformer into an ideal shunt transformer.

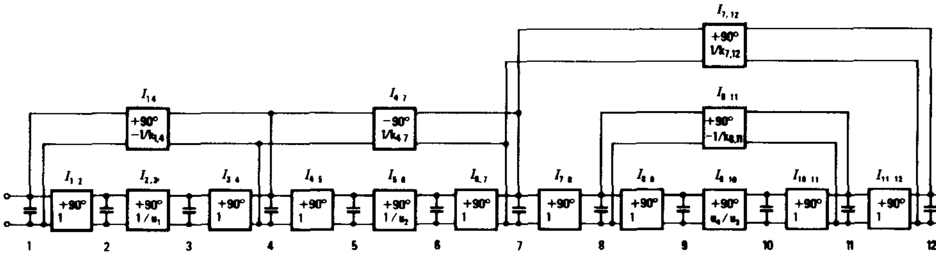

Fig. 9. Low-pass circuit configuration after the equivalent conversion by impedance inverters.

"inductive and capacitive coupling," the terms "positive and negative coupling" are to be preferred because in the present context the sign of a coupling is of more interest than its frequency response. In the case of our example, only the additional coupling of capacitors C_4 and C_7, serving for the production of the attenuation pole at $\Omega = 1.26$, is negative, while all the other direct couplings or bypass couplings are positive. Other sign distributions are readily conceivable, e.g., under otherwise unchanged circumstances the signs of the couplings $I_{7,12}$, $I_{8,11}$, $I_{9,10}$ could be negative. Further transformation of the network can cause all n shunt capacitors to be of equal value, but this conversion would be unnecessary because the coupling bandwidths

$$\Delta f_{\mu,\nu} = f_0 B \frac{|k_{\mu,\nu}|}{u_{\mu,\nu}\sqrt{a_\mu a_\nu}} \tag{15}$$

required for realizing the bandpass filter depend only on the normalized low-pass element values

$$a_1, a_2, a_3, \cdots, a_n = \begin{cases} c_1, l_2, c_3, \cdots, c_n & (n \text{ odd}) \\ c_1, l_2, c_3, \cdots, l_n & (n \text{ even}) \end{cases} \tag{16}$$

and the ideal transformers $u_{\mu,\nu}$ and $k_{\mu,\nu}$.

The coefficient $u_{\mu,\nu}$ should only be taken into account if the network contains a transformer between the low-pass elements μ and ν and, at the same time, $\nu = \mu + 1$; the $k_{\mu,\nu}$ values differ from unity only in the case of the bypass couplings. In order to obtain an equivalent circuit diagram for the narrow-band bandpass filter, a conventional reactance transformation is applied to the low-pass structure (for example, Fig. 9) whereby the shunt capacitors turn into shunt parallel resonant circuits and the impedance inverters remain unchanged. Replacing the impedance inverters—as a narrow-band approximation—by lumped elements, a final equivalent circuit diagram of the microwave bandpass filter is obtained (Fig. 10).

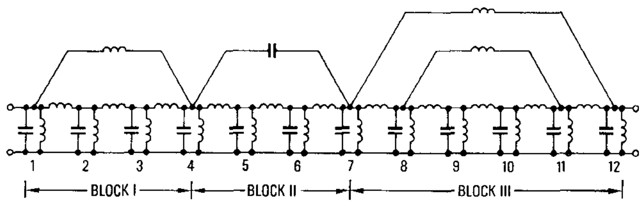

Fig. 10. Equivalent circuit diagram with lumped elements of the nonminimum-phase bandpass filter with 12 shunt resonant circuits. The additional couplings are realized in circuit blocks and consequently associated with group delay flattening and attenuation poles at finite frequencies.

V. Examples of Filters Realized

In the 1970's, some reports were published on the described method of synthesizing low-pass networks with bypassing, ideal transformers [5], [21] and on realized examples [22]–[24], [16] of steepened-attenuation and delay-equalized microwave bandpass filters with bandwidths between 25 MHz and 72 MHz. At this point, attention should be drawn to three selected examples of filters centerted at 1235 MHz and 4000 MHz which demonstrate the flexibility of the method of synthesis, the free selection of network configuration, and the close agreement between theory and practice.

The first example of realization is a 12-cavity bandpass filter with the center frequency $f_0 = 1235$ MHz and the theoretical Chebyshev bandwidth $Bf_0 = 26.45$ MHz. The low-pass network with circuit blocks, which is characterized by the pole positions (13), the chain matrix (14), and the structure shown in Fig. 9 and which was already treated in the preceding section, was chosen as the reference low-pass prototype. An equivalent circuit diagram for the microwave bandpass filter is shown in Fig. 10. To realize the Q factor of approximately 3500, the cavities of the filter were designed as coaxial resonators with a square outer conductor of the edge length 40 mm. Fig. 11 shows a photograph of the breadboard model with cover plate removed. The arrangement of the resonators as indicated in Fig. 7 assures that not only the successive resonators but also those that

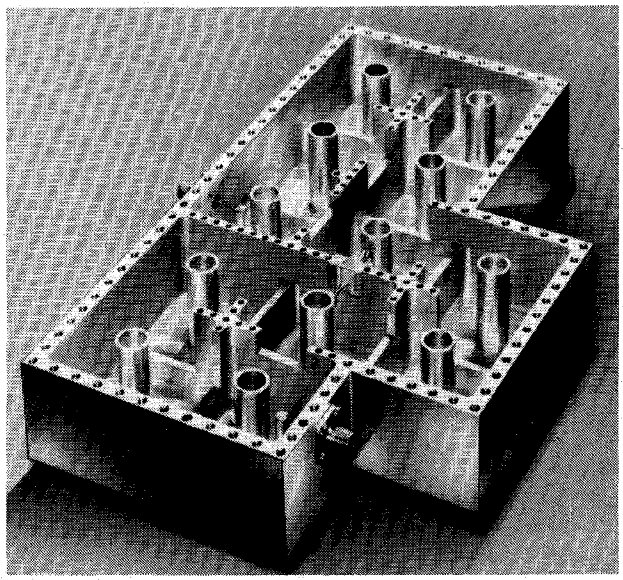

Fig. 11. Photograph of the broadboard model (cover plate removed) of the nonminimum-phase 12-resonator band-pass filter with attenuation poles at finite frequencies.

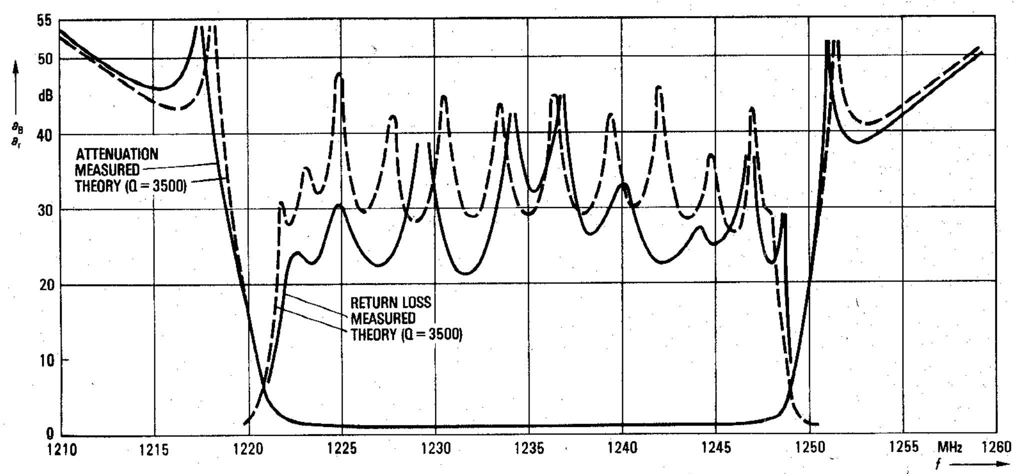

Fig. 12. Measured and computed attenuation and return loss responses of the 12-resonator coaxial bandpass filter.

are to be additionally coupled will each have a common separating wall for mounting the coupling element. Positive couplings are realized by providing inductive slots in the appropriate partitioning walls; resonators 4 and 7 are negatively coupled by means of a capacitive antenna configuration. The length of the inner conductor was chosen shorter than $\lambda/4$ in order to save space. Capacitive tuning screws plunging from the filter cover plate into the tubular inner conductor serve for tuning the coaxial resonators to the center frequency. Filter housing and cover plate are made of aluminum in the interest of low weight; the use of brass for the inner conductors and of steel for the tuning screws assure, for the present dimensions, a temperature coefficient of $|TC| \leq 1 \cdot 10^{-6}/K$. The measured responses, as well as those calculated from the equivalent network (Fig. 10), of the attenuation, the return loss, and the group delay are compared in Figs. 12 and 13 and show close agreement. The variation of the group delay over the theoretical inter-

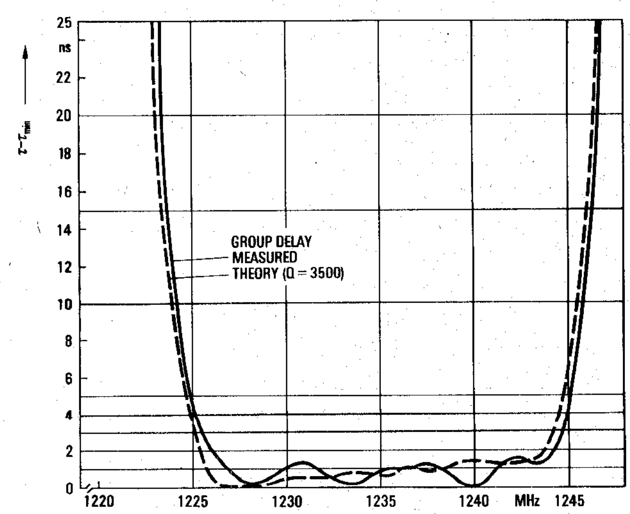

Fig. 13. Measured and computed group delay response of the 12-resonator coaxial bandpass filter.

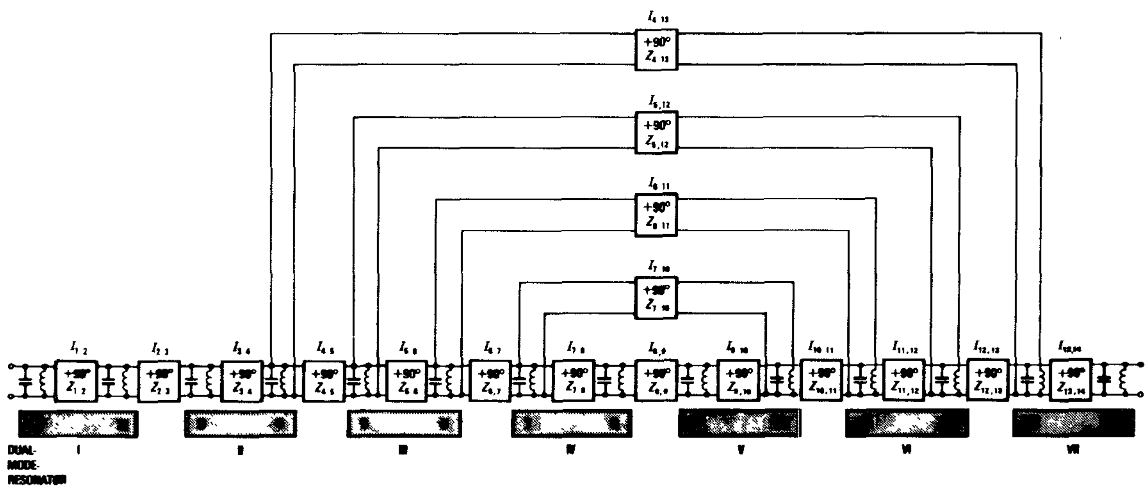

Fig. 14. Equivalent circuit diagram of the 14-resonator bandpass filter with four bypass couplings for group delay flattening. Each pair of resonators in a dual-mode cavity is identified by a solid line framing the respective resonator numbers.

val $\Omega_T Bf_0 = 17.4$ MHz (with $\Omega_T = 0.66$ as per Fig. 3 and $Bf_0 = 26.45$ MHz) is less than $\tau - \tau_{\min} = 1.5$ ns. Two well-defined attenuation poles appear at finite frequencies as a result of the additional capacitive coupling of cavities 4 and 7.

The operating frequencies of, for example, 4 GHz and 6 GHz of satellite systems make it necessary to use filters in these frequency ranges that, besides satisfying stringent electrical specifications, also have low weight and are almost insensitive to temperature. As filter modules of high quality, waveguide resonators that are usually operated in the fundamental modes, i.e., the TE_{101} mode in the resonator of rectangular cross section, and the TE_{111} mode in the circular cross-section resonator, are suitable for this application. By utilizing two orthogonal modes in the resonator of square or circular cross section it is further possible to reduce both weight and volume by 50 percent. A further considerable reduction in weight is achieved by using composite graphite fiber epoxy materials instead of thin-wall invar techniques.

W.-G. Lin [25] in 1951 and Butterweck [26] in 1961 already drew attention to microwave filters using different resonant modes in a single cavity, but it was not until 1970 that the multiple utilization of cavities was introduced in modern filter design by Williams [27]. In the interim, further examples of the realization of filters in dual-mode technology that are based on this principle have been introduced, e.g., [28]–[32].

As shown by the following example of the realization of a nonminimum-phase 14-resonator bandpass filter for 4 GHz with a usable bandwidth of 36 MHz, the proposed synthesis technique can also be successfully applied to dual-mode filters. In order to be able to satisfy the specifications for a bandpass filter, a low-pass network of the order $n = 14$ was chosen whose characteristic function $K(s)$ contains two pole quadruplets ($\alpha_1 = 0.552170$, $\beta_1 = 0.636318$, $\alpha_2 = 0.633838$, $\beta_2 = 0.205108$), and whose passband $0 \leqslant \Omega \leqslant 1$ assures the return loss $a_r \geqslant 30$ dB. No poles at finite frequencies are provided; the normalized group delay response $T(\Omega)$ is flattened by $\Delta T_{\max} \approx 0.0033$ over the frequency range $0 \leqslant \Omega \leqslant 0.715$. For a bandpass filter with a Chebyshev bandwidth of $Bf_0 = 38.5$ MHz corresponding to (2), this results in a maximum group delay ripple of $\Delta\tau_{\max} \approx 0.18$ ns within $|f - f_0| \leqslant 13.76$ MHz. Fig. 14 presents the equivalent circuit diagram of the bandpass filter (without considering the different frequency responses of slot couplings and dual-mode couplings) in which only positive couplings are present. The association of two respective resonant circuits with each dual-mode resonator is indicated by a symbol.

The realization of the bandpass filter with dual-TE_{101}-mode cavities is represented schematically in Fig. 15. It shows the position of the coupling elements and the vectors of the electrical field strength associated with the resonant circuits; further details are to be seen in the photograph of the filter (with cover plate removed) in Fig. 16. Compared with the coaxial filter, the dual-mode filter has the constraint that not only the resonant circuits to be coupled in different cavities need to have a common partitioning wall, but also that the electrical vectors of the respective resonant circuits are additionally required to have parallelity. If both resonant circuits of a cavity are coupled to the two resonant circuits of an adjacent cavity (as, for example, with resonant circuits 8 and 9 as well as 7 and 10 in cavities IV and V shown in Fig. 15), the position of the slots must assure mode decoupling. With the cavity dimensions of $48 \times 48 \times 57.5$ mm^3 preventing spurious responses at 4 GHz it is possible in practice to realize a Q factor of 9500. Since no negative couplings are present, all dual-mode coupling screws 1-2 $\cdots$ 13-14 are placed on the same side of the filter relative to the reference system of the vectors of the electrical field strength. $Bf_0 = 38.5$ MHz was chosen for theoretical Chebyshev bandwidth. Figs. 17 and 18 depict measured and computed responses of attenuation and group delay. Close agreement will be noted between the theoretical and the measured responses. The measured

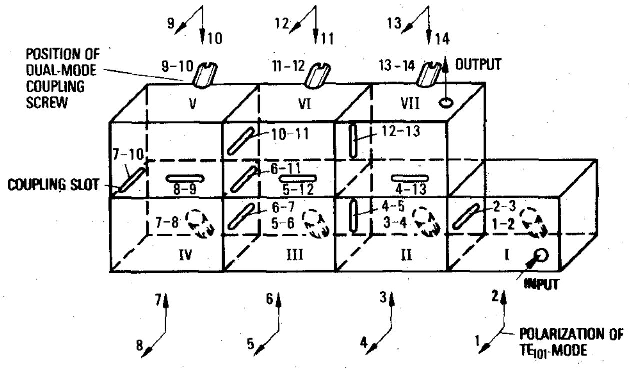

Fig. 15. Configuration of the nonminimum-phase 14-resonator bandpass filter with dual TE_{101}-mode cavities. The coupling slots, dual-mode coupling screws, filter connections, and polarization of the electrical field strengths are indicated.

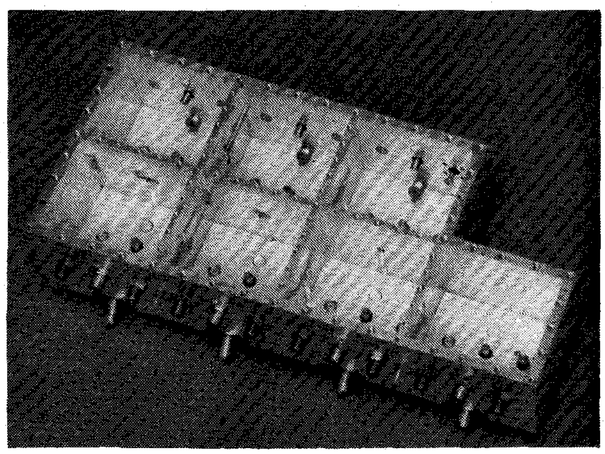

Fig. 16. Photograph of the experimental nonminimum-phase dual TE_{101}-mode 14-resonator bandpass filter. The cover plate is removed.

return loss response is greater than 21.5 dB within the passband.

As an alternative approach to the described example of realization, the nonminimum-phase 14-resonator filter with dual-mode cavities in a single-line cavity array is feasible if nonsuccessive resonant circuits are also to be combined into a dual-mode cavity. The realization scheme in Fig. 19 shows not only the correlations between dual-mode cavities and filter resonant circuits but also the position of the coupling elements and filter connectors. The single-line array design offers, besides the electrically favorable possibility of forming blocks, the advantage that it is also possible to use the technologically more easily fabricated TE_{111} circular waveguides, and that simple connecting flanges are often sufficient for connecting the cavities and diaphragms.

The universal applicability of the described synthesis technique is particulary evident in the realization of a 6-resonator elliptic bandpass filter with dual TE_{101}- or TE_{111}-mode cavities. The characteristic function of the low-pass prototype exhibits two attenuation poles at finite frequencies. Two variants of this filter were synthesized. The first has a symmetrical structure and contains additional couplings between resonant circuits 1 and 6 as well as 2 and 5 [33], while the second contains additional couplings between resonant circuits 1 and 6 as well as 1

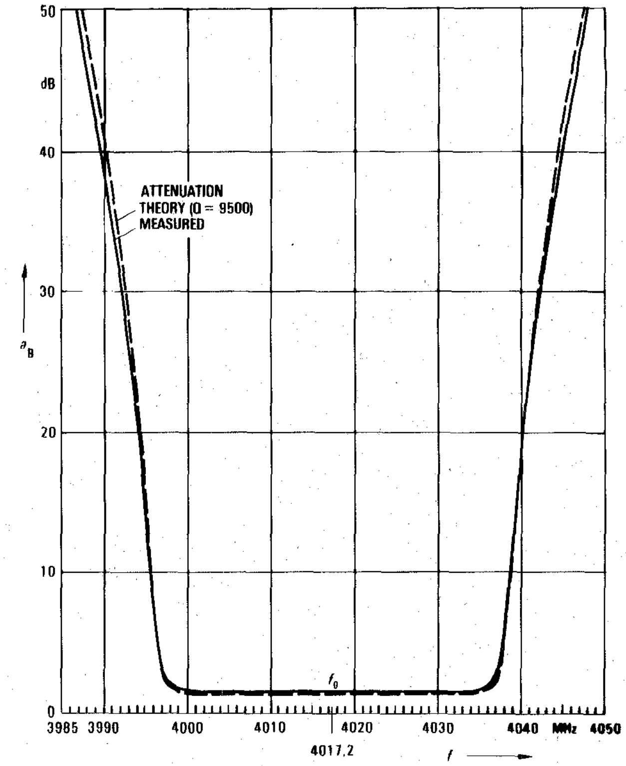

Fig. 17. Measured and computed attenuation response of the nonminimum-phase dual TE_{101}-mode 14-resonator waveguide filter.

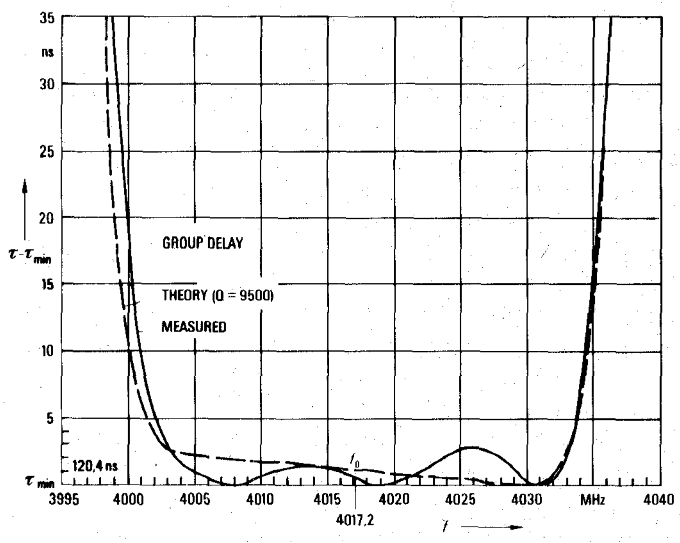

Fig. 18. Measured and computed group delay response of the nonminimum-phase dual TE_{101}-mode 14-resonator waveguide filter.

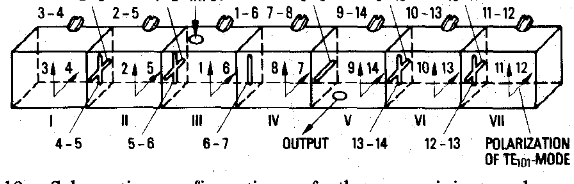

Fig. 19. Schematic configuration of the nonminimum-phase dual TE_{101}-mode 14-resonator waveguide filter, the cavities of which are arranged as a single-line array.

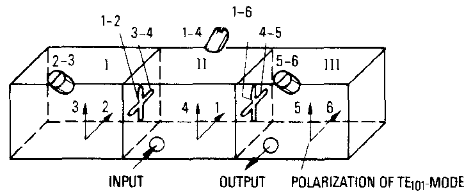

Fig. 20. Cavity arrangement of the dual TE_{101}-mode 6-resonator elliptic bandpass filter.

and 4 [23]. The flexibility of the described synthesis technique will be demonstrated with reference to the above unsymmetrical structure of the bandpass filter. The control of extracting the elements can already be effected at the design stage in such a way that the special conditions for the realization with dual-mode cavities are taken into account. All couplings are positive except for the additional coupling of resonant circuits 1 and 4. Since change of sign of a coupling can only be realized in dual-mode filters of this type by altering the position of the corresponding dual-mode coupling screw by 90° in the plane formed by the vectors of the electrical field strength, the resonant circuits 1 and 4 must be realized in the same dual-mode cavity, leading to the filter structure shown schematically in Fig. 20. The realization of such a bandpass filter for 4 GHz with a Chebyshev bandwidth of 40.36 MHz (theoretical return loss $a_r \geqslant 26$ dB) is described and its theoretical and measured curves presented in [23].

VI. Conclusions

The present study is concerned with the synthesis and realization of narrow-band microwave bandpass filters that are due to the additional couplings of nonsuccessive resonant circuits on the one hand nonminimum-phase and therefore possess degrees of freedom for delay equalization in the passband, and on the other hand have attenuation poles at finite frequencies. The realization of canonical network structures with bypass couplings is possible by extracting from the chain matrix an ideal transformer, the input of which is connected, for example, in shunt with and its output in series with the remainder two-port. The extraction is associated with the reduction by 2 of the degree of the denominator polynomial of the chain matrix elements. To realize a clear correlation between the bypass elements and the electrical performance, canonical structures with sections that are bypassed only once or twice are proposed. Examples of applications for the described synthesis of narrow-band bandpass filters whose couplings produce attenuation poles at finite frequencies or group delay equalization in the passband are described. The theoretical and measured curves of the examples of realization for 1235 MHz, consisting of coaxial cavities, and for 4000 MHz, consisting of dual-mode cavities, are seen to be in close agreement.

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Dual-Mode Dielectric Resonator Loaded Cavity Filters

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Abstract —**A new miniature realization of dual-mode filters is presented. As a basic building element of the filter, the dielectric resonator axially mounted in a waveguide below cutoff and resonating in a hybrid, degenerate mode is used. A dramatic reduction in size and weight, and excellent temperature performance with minimum degradation of resonator Q was achieved. Dual-mode configuration, preferred in satellite applications, allows simple realization of high-performance elliptic function filters. Experimental results are presented and demonstrate excellent agreement with the theory. Bandpass filter configuration is discussed; however, realizations of bandstop, directional, etc., filters are also possible.**

I. Introduction

TO EFFICIENTLY utilize an allocated frequency spectrum, channelization is usually necessary. In a typical communication system, this is accomplished by the use of a large number of high-performance bandpass filters. Due to narrow bandwidth and required low loss in the passband of individual filters, high Q waveguide cavities are used. However, such filters are bulky and present sharp contrast when compared to other microwave components especially those using MIC technology.

In communication satellite applications, waveguide filters impose severe constraints as far as the weight and volume of a communication transponder is concerned. At present, to reduce weight, two implementations of cavity filters are used; thin-wall INVAR and graphite fiber reinforced plastic (GFRP) technologies. A dual-mode approach pioneered by Atia and Williams [1] (using degenerate cavity modes) can be used to realize conveniently high performance, elliptic function filters requiring coupling between nonadjacent cavities. However, even such advanced filters present a major constraint in satellite layout, and further reduction in size and weight is still needed.

Due to recent developments in ceramics technology and

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